Inversa de una matriz usando el método de Gauss

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Calcula por el método de Gauss la inversa de la matriz: $A = \begin{pmatrix} -3 & 2 & -1 \\ -4 & -1 & -2 \\ 3 & 1 & 0 \end{pmatrix}$

Solución

Escribamos la matriz ampliada y usemos Gauss:

$$\left(\begin{array}{cccccc} -3 & 2 & -1 & 1 & 0 & 0 \\ -4 & -1 & -2 & 0 & 1 & 0 \\ 3 & 1 & 0 & 0 & 0 & 1 \end{array}\right) \left\{\begin{array}{ccc} F_1 & \leftrightarrows & -F_1+F_2 \\ F_2 & \leftrightarrows & F_2 \\ F_3 & \leftrightarrows & F_3 \end{array}\right\} \longmapsto$$

$$\left(\begin{array}{cccccc} -1 & -3 & -1 & -1 & 1 & 0 \\ -4 & -1 & -2 & 0 & 1 & 0 \\ 3 & 1 & 0 & 0 & 0 & 1 \end{array}\right) \left\{\begin{array}{ccc} F_1 & \leftrightarrows & F_1 \\ F_2 & \leftrightarrows & -4F_1+F_2 \\ F_3 & \leftrightarrows & 3F_1+F_3 \end{array}\right\} \longmapsto$$

$$\left(\begin{array}{cccccc} -1 & -3 & -1 & -1 & 1 & 0 \\ 0 & 11 & 2 & 4 & -3 & 0 \\ 0 & -8 & -3 & -3 & 3 & 1 \end{array}\right) \left\{\begin{array}{ccc} F_1 & \leftrightarrows & F_1 \\ F_2 & \leftrightarrows & F_3 \\ F_3 & \leftrightarrows & F_2 \end{array}\right\} \longmapsto$$

$$\left(\begin{array}{cccccc} -1 & -3 & -1 & -1 & 1 & 0 \\ 0 & -8 & -3 & -3 & 3 & 1 \\ 0 & 11 & 2 & 4 & -3 & 0 \end{array}\right) \left\{\begin{array}{ccc} F_1 & \leftrightarrows & F_1 \\ F_2 & \leftrightarrows & 4F_2+3F_3 \\ F_3 & \leftrightarrows & F_3 \end{array}\right\} \longmapsto$$

$$\left(\begin{array}{cccccc} -1 & -3 & -1 & -1 & 1 & 0 \\ 0 & 1 & -6 & 0 & 3 & 4 \\ 0 & 11 & 2 & 4 & -3 & 0 \end{array}\right) \left\{\begin{array}{ccc} F_1 & \leftrightarrows & F_1 \\ F_2 & \leftrightarrows & F_2 \\ F_3 & \leftrightarrows & -11F_2+F_3 \end{array}\right\} \longmapsto$$

$$\left(\begin{array}{cccccc} -1 & -3 & -1 & -1 & 1 & 0 \\ 0 & 1 & -6 & 0 & 3 & 4 \\ 0 & 0 & 68 & 4 & -36 & -44 \end{array}\right) \left\{\begin{array}{ccc} F_1 & \leftrightarrows & F_1 \\ F_2 & \leftrightarrows & F_2 \\ F_3 & \leftrightarrows & \dfrac{F_3}{4} \end{array}\right\} \longmapsto$$

$$\left(\begin{array}{cccccc} -1 & -3 & -1 & -1 & 1 & 0 \\ 0 & 1 & -6 & 0 & 3 & 4 \\ 0 & 0 & 17 & 1 & -9 & -11 \end{array}\right) \left\{\begin{array}{ccc} F_1 & \leftrightarrows & F_1 \\ F_2 & \leftrightarrows & F_2 \\ F_3 & \leftrightarrows & \dfrac{F_3}{17} \end{array}\right\} \longmapsto$$

$$\left(\begin{array}{cccccc} -1 & -3 & -1 & -1 & 1 & 0 \\ 0 & 1 & -6 & 0 & 3 & 4 \\ 0 & 0 & 1 & 1/17 & -9/17 & -11/17 \end{array}\right) \left\{\begin{array}{ccc} F_1 & \leftrightarrows & F_1+F_3 \\ F_2 & \leftrightarrows & F_2+6F_3 \\ F_3 & \leftrightarrows & F_3 \end{array}\right\} \longmapsto$$

$$\left(\begin{array}{cccccc} -1 & -3 & 0 & -16/17 & 8/17 & -11/17 \\ 0 & 1 & 0 & 6/17 & -3/17 & 2/17 \\ 0 & 0 & 1 & 1/17 & -9/17 & -11/17 \end{array}\right) \left\{\begin{array}{ccc} F_1 & \leftrightarrows & F_1+3F_2 \\ F_2 & \leftrightarrows & F_2 \\ F_3 & \leftrightarrows & F_3 \end{array}\right\} \longmapsto$$

$$\left(\begin{array}{cccccc} -1 & 0 & 0 & 2/17 & -1/17 & -5/17 \\ 0 & 1 & 0 & 6/17 & -3/17 & 2/17 \\ 0 & 0 & 1 & 1/17 & -9/17 & -11/17 \end{array}\right) \left\{\begin{array}{ccc} F_1 & \leftrightarrows & -F_1 \\ F_2 & \leftrightarrows & F_2 \\ F_3 & \leftrightarrows & F_3 \end{array}\right\} \longmapsto$$

$$\left(\begin{array}{cccccc} 1 & 0 & 0 & -2/17 & 1/17 & 5/17 \\ 0 & 1 & 0 & 6/17 & -3/17 & 2/17 \\ 0 & 0 & 1 & 1/17 & -9/17 & -11/17 \end{array}\right)$$

Por tanto:

$$A^{-1} = \begin{pmatrix} -2/17 & 1/17 & 5/17 \\ 6/17 & -3/17 & 2/17 \\ 1/17 & -9/17 & -11/17 \end{pmatrix}$$