Cálculo de límites usando la regla de L'Hopital (para algunas indeterminaciones)

Ejercicio. Calcule

1. $\lim_{x\to 0}\left(\frac{-2x+e^x-e^{-x}}{x-\sin(x)}\right)$
2. $\lim_{x\to\infty}\left(\frac{x}{2x+\log(x)^3}\right)$
3. $\lim_{x\to 0}\left(\frac{e^x-e^{-x}}{\sin(x)}\right)$
4. $\lim_{x\to 0}\left(\frac{\log(x+1)-\sin(x)}{x\sin(x)}\right)$
5. $\lim_{x\to 0}\left(\frac{-x+\sin(x)}{x\sin(x)}\right)$
6. $\lim_{x\to\frac{\pi}{4}}\left(\frac{\tan(x)-1}{\cos(2x)}\right)$
7. $\lim_{x\to 0^+}\left(\frac{e^{\frac{1}{x}}}{\log(x)}\right)$
8. $\lim_{x\to 0}\left(\frac{e^x-e^{\sin(x)}}{x^3}\right)$
9. $\lim_{x\to 0}\left(\frac{-2x+\cos(x)-e^{-2x}}{\sin^2(x)}\right)$
10. $\lim_{x\to 1}\left(\frac{1-\cos(2\pi x)}{(x-1)^2}\right)$

Solución

1. $\lim_{x\to 0}\left(\frac{-2x+e^x-e^{-x}}{x-\sin(x)}\right)$ = Indt $\left[\frac{0}{0}\right]$ L'Hôpital = $\lim_{x\to 0}\left(\frac{e^x-2+e^{-x}}{1-\cos(x)}\right)=$ Indt $\left[\frac{0}{0}\right]$ L'Hôpital = $\lim_{x\to 0}\left(\frac{e^x-e^{-x}}{\sin(x)}\right)$ = Indt $\left[\frac{0}{0}\right]$ L'Hôpital = $\lim_{x\to 0}\left(\frac{e^x+e^{-x}}{\cos(x)}\right)=$ 2
2. $\lim_{x\to\infty}\left(\frac{x}{2x+\log(x)^3}\right)$ = Indt $\left[\frac{\infty}{\infty}\right]$ L'Hôpital = $\lim_{x\to\infty}\frac{1}{2+\frac{3\log(x)^2}{x}}=\lim_{x\to\infty}\left(\frac{x}{2x+3\log(x)^2}\right)$ = Indt $\left[\frac{\infty}{\infty}\right]$ L'Hôpital = $\lim_{x\to\infty}\frac{1}{2+\frac{6\log(x)}{x}}=\lim_{x\to\infty}\left(\frac{x}{2x+6\log(x)}\right)$ = Indt $\left[\frac{\infty}{\infty}\right]$ L'Hôpital = $\lim_{x\to\infty}\frac{1}{2+\frac{6}{x}}=\lim_{x\to\infty}\left(\frac{x}{2x+6}\right)$ = Indt $\left[\frac{\infty}{\infty}\right]$ L'Hôpital = $\lim_{x\to\infty}\frac{1}{2}=\frac{1}{2}$
3. $\lim_{x\to 0}\left(\frac{e^x-e^{-x}}{\sin(x)}\right)$ = Indt $\left[\frac{0}{0}\right]$ L'Hôpital = $\lim_{x\to 0}\left(\frac{e^x+e^{-x}}{\cos(x)}\right)=2$

4. $\lim_{x\to 0}\left(\frac{\log(x+1)-\sin(x)}{x\sin(x)}\right)=\text{Indt}\left[\frac{0}{0}\right]\text{ L'Hôpital}=\lim_{x\to 0}\left(\frac{-\cos(x)+\frac{1}{x+1}}{x\cos(x)+\sin(x)}\right)$
$=\text{Indt}\left[\frac{0}{0}\right]\text{ L'Hôpital}=\lim_{x\to 0}\left(\frac{\sin(x)-\frac{1}{(x+1)^2}}{-x\sin(x)+2\cos(x)}\right)=-\frac{1}{2}$

5. $\lim_{x\to 0}\left(\frac{-x+\sin(x)}{x\sin(x)}\right)=\text{Indt}\left[\frac{0}{0}\right]\text{ L'Hôpital}=\lim_{x\to 0}\left(\frac{\cos(x)-1}{x\cos(x)+\sin(x)}\right)$
$=\text{Indt}\left[\frac{0}{0}\right]\text{ L'Hôpital}=\lim_{x\to 0}\left(-\frac{\sin(x)}{-x\sin(x)+2\cos(x)}\right)=0$

6. $\lim_{x\to \frac{\pi}{4}}\left(\frac{\tan(x)-1}{\cos(2x)}\right)=\text{Indt}\left[\frac{0}{0}\right]\text{ L'Hôpital}=\lim_{x\to \frac{\pi}{4}}\left(\frac{-\tan^2(x)-1}{2\sin(2x)}\right)=$
-1

7. $\lim_{x\to 0^+}\left(\frac{e^{\frac{1}{x}}}{\log(x)}\right)=\text{Indt}\left[\frac{\infty}{\infty}\right]\text{ L'Hôpital}=\lim_{x\to 0^+}\left(-\frac{e^{\frac{1}{x}}}{x}\right)=\lim_{x\to 0^+}\left(-\frac{e^{\frac{1}{x}}}{x}\right)=$
$-\infty$

8. $\lim_{x\to 0}\left(\frac{e^x-e^{\sin(x)}}{x^3}\right)=\text{Indt}\left[\frac{0}{0}\right]\text{ L'Hôpital}=\lim_{x\to 0}\left(\frac{e^x-e^{\sin(x)}\cos(x)}{3x^2}\right)=$
$\text{Indt}\left[\frac{0}{0}\right]\text{ L'Hôpital}=\lim_{x\to 0}\left(\frac{e^x+e^{\sin(x)}\sin(x)-e^{\sin(x)}\cos^2(x)}{6x}\right)=\text{Indt}$
$\left[\frac{0}{0}\right]\text{ L'Hôpital}=\lim_{x\to 0}\left(\frac{e^x}{6}+\frac{e^{\sin(x)}\sin(x)\cos(x)}{2}-\frac{e^{\sin(x)}\cos^3(x)}{6}+\frac{e^{\sin(x)}\cos(x)}{6}\right)=$
$\frac{1}{6}$

9. $\lim_{x\to 0}\left(\frac{-2x+\cos(x)-e^{-2x}}{\sin^2(x)}\right)=\text{Indt}\left[\frac{0}{0}\right]\text{ L'Hôpital}=\lim_{x\to 0}\left(\frac{-\sin(x)-2+2e^{-2x}}{2\sin(x)\cos(x)}\right)$
$=\text{Indt}\left[\frac{0}{0}\right]\text{ L'Hôpital}=\lim_{x\to 0}\left(\frac{-\cos(x)-4e^{-2x}}{-2\sin^2(x)+2\cos^2(x)}\right)=-\frac{5}{2}$

10. $\lim_{x\to 1}\left(\frac{1-\cos(2\pi x)}{(x-1)^2}\right)=\text{Indt}\left[\frac{0}{0}\right]\text{ L'Hôpital}=\lim_{x\to 1}\left(\frac{2\pi\sin(2\pi x)}{2x-2}\right)=$
$\text{Indt}\left[\frac{0}{0}\right]\text{ L'Hôpital}=\lim_{x\to 1}\left(2\pi^2\cos(2\pi x)\right)=2\pi^2$

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